A PHYSICS ALPHABET

ABBREVIATIONS USED IN THIS BOOK, UNITS AND FORMULAE[1]

Quantity (with symbol)		Unit
Area (e.g. surface area)	A	m^2
Acceleration	a	m/s^2
Specific heat capacity	c	$J/(kg\,°C)$
Critical angle	i_c	°
Speed of light	c	m/s
Energy	E	J = Nm
Force	F	N = kg m/s^2
Frequency	f	Hz = 1/s
Focal length (of lens)	f	m
Gravitational field strength	g	m/s^2 or N/kg
Height	h	m
Current	I	A
Angle of incidence	i	°
Intensity	I	W/m^2
Spring constant	k	N/m
Specific latent heat	L	J/kg
Mass	m	kg
Number of turns on coil	N	
Refractive index	n	
Momentum	p	kg m/s or Ns

Quantity (with symbol)		Unit
Pressure	p	Pa = N/m^2
Power	P	W = J/s
Power (of lens)	P	D = 1/m
Charge	Q	C = As
Radius	r	m
Resistance	R	Ω = V/A
Angle of refraction	r	°
Distance or displacement	s	m
Time	t	s
Temperature	T	°C or K
Time period	T	s
Object distance (lens)	u	m
Speed before change	u	m/s
Speed or velocity	v	m/s
Image distance (lens)	v	m
Voltage	V	V = J/C
Volume	V	m^3
Weight	W	N = kg m/s^2
Extension	x	m
Wavelength	λ	m
Density	ρ	kg/m^3

1 km = 1000 m	1 Mm = 10^6 m	1 Gm = 10^9 m	
1 cm = 0.01 m	1 mm = 0.001 m	1 µm = 10^{-6} m	1 nm = 10^{-9} m

Units with powers: note 1 cm^2 means 1 cm × 1 cm = 0.01 m × 0.01 m = 10^{-4} m^2

[1] A list of formulae and data is given on the inside back cover.

FORMULAE AND DATA[2]

The meaning of all symbols in the formulae, and the units used, are given on the inside of the cover. If you need to revise a formula, turn to the page listed alongside it in this table.

Velocity and Acceleration	
$s = vt$	P 20
$v - u = at$	P 27
Force and Acceleration	
$F = ma$	P 36
Weight	
$W = mg$	P 38
Pressure	
$p = F/A$	P 48
$p = \rho g h$	P 50
Momentum	
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$p_{\text{after}} - p_{\text{before}} = Ft$	P 58
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$a = v^2/r$	P 52
$F = mv^2/r$	P 52
Springs and Elastic Deformation	
$F = kx$	P 113
$E = \frac{1}{2}kx^2$	P 115
Energy and Power	
$E = Pt$	P 100
Energy or Work Done	
$E = Fs$	P 100
Kinetic Energy (motion energy)	
$E = \frac{1}{2}mv^2$	P 104
Gravitational Potential Energy	
$E = mgh$	P 101
Energy and Temperature change	
$\Delta Q = mc\Delta T$	P 92
Energy and Change of State	
$Q = mL$	P 95

Efficiency	
$\text{efficiency} = \frac{\text{useful energy transferred}}{\text{total energy transferred}}$	P 107
Electricity	
$Q = It$	P 66
$V = IR$	P 73
$P = IV$	P 80
$P = V^2/R = I^2R$	P 82
$E = Pt = IVt$	P 80
Transformers	
$\frac{V_s}{V_p} = \frac{N_s}{N_p}$	P 88
Oscillations	
$f = 1/T$	P 117
Waves	
$v = f\lambda$	P 117
Intensity	
$I = P/A = P/(4\pi r^2)$	P 151
Refractive Index	
$n = c/v$	P 138
Refraction (Snell's Law)	
$n_1 \sin(i) = n_2 \sin(r)$	P 140
Critical Angle	
$\sin(i_c) = 1/n$	P 142
Lenses	
$P = 1/f$	P 145
$1/v = 1/f - 1/u$	P 146
Gases	
$\frac{p_{\text{after}} V_{\text{after}}}{T_{\text{after}}} = \frac{p_{\text{before}} V_{\text{before}}}{T_{\text{before}}}$	P 180
Equivalence of Energy and Mass	
$E = mc^2$	P 165

In the questions on these worksheets, take

- **Gravitational field strength (g) as 10 N/kg**
- **Acceleration of a dropped object on Earth without air resistance (g) as 10 m/s^2**
- **Speed of light (c) as 3×10^8 m/s**

Other data will be given on each worksheet when you need it.

[2] A list of quantities, symbols and data is given on the inside front cover.

Isaac Physics Skills

Mastering GCSE Physics

A.C. Machacek & K.O. Dalby
Westcliff High School for Boys

with extra questions written by R. Meikle

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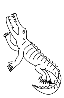
Use this collection of worksheets in parallel with the electronic version at `isaacphysics.org`. Marking of answers and compilation of results is free on Isaac Physics. Register as a student or as a teacher to gain full functionality and support.

 used with kind permission of M. J. Rutter.

Notes for the Student and the Teacher

Some students (and teachers) may seem daunted by the idea of performing calculations on momentum conservation, lens images, circular motion, nuclear energy and the like, below the age of 16. However, it is our conviction that all secondary school students can gain mastery of these concepts. By mastery, we mean that you feel confident that you understand them and can apply them to reasonably straightforward problems with accuracy. To begin, all you need is the ability to multiply and divide numbers (using a calculator), the willingness to give Physics a go, and the determination not to listen to any thoughts which say that it’s going to be too difficult.

Each worksheet contains notes, explanations, worked examples and then questions. It is important that you can see where formulae come from, and accordingly explanations in places will go deeper than required for GCSE.

Once you have read the notes, you are ready to try the questions, they get harder as you go on. You can check your answers using the isaacphysics.org website.

We suggest that you revisit each sheet and its questions until you can answer at least three quarters of the material correctly; see the pass mark indicated in the square on each sheet. Until 75% is achieved, study further, then repeat a selection of questions. This is the mastery method here ensuring a good foundation is laid for a GCSE physics education. $^{x}/_{y}$

You may well also find this book helpful when you come to revise. When you revise, resist the temptation to move beyond a page until you have attempted a good selection of the questions and have got at least three-quarters of them correct. Be aware that this book is not written with a particular specification in mind - not all sections will be relevant to your exam, especially those marked ♡. A specification map is on isaacphysics.org. ♡

Also remember, this is a Physics book not an exam revision guide. Its aim is to help you understand the principles. This may well take longer than memorizing a few soundbites, but, once achieved, enables you to solve a wide range of problems with very little further ‘learning’. Quite simply, as thousands of our students have found, once you ’get it’ it is then ‘obvious’ and no further notes or books are needed. This is mastery - and this is what you should aim for.

ACM & KOD
Westcliff-on-Sea, 2016

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We must also thank Rob Meikle for his generosity in allowing us to use many of his excellent Physics questions from his rich resource 'Physics Examples' within many of the worksheets.

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Finally, we thank you, the student. We wish you well in your studies, and trust that with a bit of determination, you will use these to help you become a master of physics at GCSE level, and that this might encourage you to try more physics in the sixth form.

Soli Deo Gloria,

ACM & KOD
Westcliff-on-Sea, 2016

Suggestions for use in lessons

Traditional approach

- Introduce the concept you wish to teach – perhaps by giving an example of a situation where this is going to be useful in the solution. A problem could be set, or a short video of a relevant situation shown (I like showing a YouTube clip of a North American 'Fall' festival in which a 500 kg pumpkin is dropped on an old school bus, when about to discuss gravitational and kinetic energy, or the Mythbusters video in which a compressed gas cylinder bursts through a breeze block wall when its regulator is sheared off, as an introduction to gas pressure).
- If desired, you can project the relevant page from the teacher's section of IsaacPhysics. This includes notes with essential details, but with some key words and definitions missing, and spaces for certain explanations. Before students have opened their Isaac books, teach the main concepts, give the definitions and use class questioning and discussion to agree the answers to the 'cloze text' parts.
- Students then turn to the relevant page, and read the 'notes' section. Students may make their own notes in their exercise books if you wish.
- While many students will be ready to begin the questions straight away (and will be able to do so without further help from you), others will need your specific help in going over important points in the notes.
- By the time that the students who needed your help with the notes are ready to start the more straightforward questions, the others will have reached the more tricky questions, and will wish assistance. Questions can be answered in students' exercise books, showing working, with the final answer to an appropriate number of significant figures and with a unit. You may choose to make certain questions optional.
- Follow-up questions can be selected from the Isaac Physics website for homework.

Use with 'Flipped Lessons'

- Here, you would set a homework to study the notes of a particular page, and complete some of the more straightforward questions – you can check their progress using isaacphysics.org.

- Students then work on questions in class, as above. The teachers' version of the text (with spaces for the explanations and results of class discussion) could be projected onto the screen and discussed as the starter for the lesson to see how much students remember and understand from their own reading.

Using Isaac Physics with this book

Isaac Physics offers online versions of each sheet at:

`isaacphysics.org/gcsebook`

There, a student can enter answers as well as learn the concepts detailed in these worksheets by reading the online versions. This online tool will mark answers, giving immediate feedback to a student who, if registered on isaacphysics.org, can have their progress stored and even retrieved for their CV! Teachers can set a sheet for class homework as the appropriate theme is being taught, and again for pre-exam revision. Isaac Physics can return the fully assembled and analysed marks to the teacher, if registered for this free service. Isaac Physics zealously follows the significant figures (sf) rules and warns if your answer has a sf problem.

Uncertainty and Significant Figures

In physics, numbers represent measurements that have uncertainty and this is indicated by the number of significant figures in an answer.

Significant figures

When there is a decimal point (dp), all digits are significant, except leading (leftmost) zeros: 2.00 (3 sf); 0.020 (2 sf); 200.1 (4 sf); 200.010 (6 sf)

Numbers without a dp can have an *absolute accuracy*: 4 people; 3 electrons.

Some numbers can be ambiguous: 200 could be 1, 2 or 3 sf (see below). Assume such numbers have the same number of sf as other numbers in the question.

Combining quantities

Multiplying or dividing numbers gives a result with a number of sf equal to that of the number with the smallest number of sf:

$x = 2.31$, $y = 4.921$ gives $xy = 11.4$ (3 sf, the same as x).

An absolutely accurate number multiplied in does not influence the above.

Standard form

Online, and sometimes in texts, one uses a letter 'x' in place of a times sign and ˆ denotes "to the power of":

$1\,800\,000$ could be 1.80x10ˆ6 (3 sf) and $0.000\,015\,5$ is 1.55x10ˆ-5

(standardly, 1.80×10^6 and 1.55×10^{-5})

The letter 'e' can denote "times 10 to the power of": 1.80e6 and 1.55e-5.

Significant figures in standard form

Standard form eliminates ambiguity: In $n.nnn \times 10^n$, the numbers before and after the decimal point are significant:

$191 = 1.91 \times 10^2$ (3 sf); 191 is $190 = 1.9 \times 10^2$ (2 sf); 191 is $200 = 2 \times 10^2$ (1 sf).

Answers to questions

In these worksheets and online, give the appropriate number of sf:

For example, when the least accurate data in a question is given to 3 significant figures, then the answer should be given to three significant figures; see above.

Too many sf are meaningless; giving too few discards information. Exam boards require consistency in sf, so it is best to get accustomed to proper practices.

Contents

Skills

1 Units

In Physics, measurable quantities usually have a number and a unit. The unit gives an indication of the size of that quantity and also information about what the quantity physically represents. This is best understood with examples.

A quantity such as 15 metres is clearly a length; one cannot measure a mass or a time in metres. 15 metres is a shorter length than 15 miles, but a longer length than 15 inches. Without the inclusion of a unit, a length of 15 is meaningless.

To facilitate global collaboration in science, seven units have been selected as the standard that all scientists should use. These are called SI base units (which comes from the French name: Système International d'unités). At GCSE Physics level, you are expected to know and be able to use the first six of these units.

Quantity	Unit name	Unit symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

SI derived units are units given in terms of the SI base units. A speed, for example, is always a length divided by a time. In SI derived units, a speed should be given in metres per second (m/s). A volume always includes the product of three lengths so, in SI derived units, a volume should be given in cubic metres (m^3).

You can work out what the appropriate unit for any quantity is by considering the quantities that are combined in any equation for that quantity.

Units may also include a prefix. These are included between the number and the unit and tell you by how much the number should be multiplied.

Prefix	Multiply By
mega (M)	1 000 000
kilo (k)	1 000
centi (c)	0.01
milli (m)	0.001
micro (μ)	0.000 001
nano (n)	0.000 000 001

1.1 Complete the table below with the correct SI derived units.

Quantity	Equation	Unit in terms of SI base units
Area	$A = L^2$	(a)
Acceleration	$a = (v - u)/t$	(b)
Momentum	$p = mv$	(c)
Kinetic energy	$E = \frac{1}{2}mv^2$	(d)
Gravitational potential energy	$E = mgh$	(e)
Electric charge	$Q = It$	(f)

1.2 Write the following quantities with the appropriate unit and prefix

0.000 001 20 m	(a)	5 200 000 mg	(b)
6 500 μs	(c)	0.000 000 920 km	(d)
3 400 000 nA	(e)	0.000 027 0 kA	(f)
5 500 000 000 nm	(g)	6 500 000 cm^2	(h)
0.000 044 0 km/s	(i)	83 000 mm^3	(j)

1.3 Convert these measurements to metres (m):

(a) 240 cm (b) 1 500 cm (c) 95 cm (d) 7.0×10^3 cm

1.4 Convert these mass measurements into kilograms (kg):

(a) 2 500 g (b) 350 g (c) 1 020 g (d) 3.80×10^4 g

1.5 Convert these mass measurements into grams (g):

(a) 6.70 kg (b) 3 400 mg (c) 0.050 kg (d) 150 mg

1.6 Convert the following volumes into cubic metres (m^3) [1 cm^3 = 1 ml]:

(a) 2 500 cm^3 (b) 68 cm^3 (c) 3 700 litres

1.7 Convert the following volumes to litres (L):

(a) 2 500 cm^3 (b) 2.0 m^3 (c) 560 cm^3

1.8 How many cubic centimetres (cm^3) are there in these volumes?

(a) 1.60 litres (b) 3.25 m^3 (c) 0.0625 m^3 (d) 0.080 litres

1.9 Convert these areas into square metres (m^2):

(a) 4 250 cm^2 (b) 5.3×10^4 cm^2 (c) 2.50 km^2 (d) 15.0 cm^2

1.10 Calculate the number of square centimetres (cm^2) in:

(a) 1.44 m^2 (b) 0.0275 m^2 (c) 3.50×10^{-2} m^2 (d) 1.50×10^{-4} m^2

Additional Units Questions

1.11 Change these times into seconds (s):

(a) 3.0 mins (b) 2 hrs 30 mins (c) 3.6 mins (d) 4 mins 30 secs

1.12 How many seconds are there in a minute, an hour, a day and a year?

1.13 Write the following fundamental constants and data without unit prefixes.

(a) speed of light = 300 Mm/s (b) g = 9 810 mN/kg
(c) Earth's radius = 6 370 km (d) red wavelength = 680 nm

1.14 The light-year (ly) is a unit often mistaken as a unit of time. It is defined as the distance travelled by light in a vacuum in one Julian year (365.25 days). Use the data in Q1.13 and the equation speed = distance/time ($v = s/t$). What SI measurement is 1.0 ly equivalent to?

2 Standard form

The radius of the Earth is $6\,400\,000$ m.
The speed of light is $300\,000\,000$ m/s.
The charge of one electron is $-0.000\,000\,000\,000\,000\,000\,16$ C.

Big and small numbers are inconvenient to write down – scientists and engineers use standard form to make things clearer.

The above numbers in standard form look like this:

$$6.4 \times 10^{6} \text{ (or } 6.4\,\text{e}\,6 \text{ on a computer).}$$

$$3.0 \times 10^{8} \text{ (or } 3.0\,\text{e}\,8 \text{ on a computer).}$$

$$-1.6 \times 10^{-19} \text{ (or } -1.6\,\text{e}\,-19 \text{ on a computer).}$$

number in standard form = mantissa × power of ten

The mantissa is a number bigger than or equal to 1, but less than 10.

2.1 Which of the following numbers could be a mantissa?

(a) 9.5 – *Yes, it is larger than (or equal to)* 1 *and smaller than* 10

(b) 0.4 – *No, it is smaller than* 1

(c) 12.3 – *No, it is not less than* 10

(d) 0.2

(e) 1.2

(f) 1.0

(g) 10.3

(h) 0.04

(i) 10

(j) 5

(k) 7.6

Powers of ten are numbers you can make by starting with 1 and either multiplying or dividing as many times as you like by 10.

So 100, 0.01, $100\,000$, 10, 1 and $0.000\,1$ are all powers of ten,
but 30, 0.98 and $40\,000$ are not powers of ten.

Powers of ten can be written using exponents (e.g. 10^2 rather than 100).

$10\,000 = 10 \times 10 \times 10 \times 10 = 10^4$	exponent $= 4$
$1\,000 = 10 \times 10 \times 10 = 10^3$	exponent $= 3$
$100 = 10 \times 10 = 10^2$	exponent $= 2$
$10 = 10 = 10^1$	exponent $= 1$
$1 = 10^0$	exponent $= 0$
$0.1 = 1/10 = 1/10^1 = 10^{-1}$	exponent $= -1$
$0.01 = 1/100 = 1/10^2 = 10^{-2}$	exponent $= -2$
$0.001 = 1/1000 = 1/10^3 = 10^{-3}$	exponent $= -3$

2.2 For the following numbers, decide if they are powers of ten and, if they are, write down the exponent. The first two have been done for you.

Number	Power of Ten	Exponent
30	✗	
100	✓	2
0.004	(a)	
0.01	(b)	
1 000 000 000	(c)	
0.000 000 1	(d)	

Note that the exponent counts the number of times the decimal point must be moved to get from its starting point before the number is turned into a mantissa:

e.g. $0.\underset{\rightarrow}{0}\underset{\rightarrow}{0}\underset{\rightarrow}{0}\underset{\rightarrow}{3}2 = 3.2 \times 10^{-4}$, as the decimal point must be moved 4 times to the right before it makes a 3.2.

Also $893 = 8\underset{\leftarrow}{9}\underset{\leftarrow}{3}.0 = 8.93 \times 10^2$ as the decimal point must be moved 2 times to the left before it makes an 8.93.

2.3 For each of the following numbers state how many times the decimal point must be moved (+ve to the left, −ve to the right) when making the numbers into a mantissa and the exponent of the 10 when in standard form.

(a) 0.000 145 (c) 345 094 (e) 69 023 (g) 0.011 2

(b) 153.034 2 (d) 0.003 425 39 (f) 0.000 002 87 (h) 56 920.142 2

2.4 Write the following numbers as mantissa × power of ten, then write the exponent, and finally write them in standard form. The first two are done as examples.

Number	Mantissa × Power of 10	Standard Form
450 000	$4.5 \times 100\,000$	4.5×10^{5}
0.000 032	$3.2 \times 0.000\,01$	3.2×10^{-5}
300	(a)	(b)
0.026	(c)	(d)
390 000	(e)	(f)
6 700	(g)	(h)
0.000 000 062	(i)	(j)

2.5 Express the following numbers in standard form:

(a) 4 000
(b) 0.030
(c) 8.31
(d) 0.000 002 8
(e) 860 000
(f) 0.002 451
(g) 920
(h) 0.109 3
(i) 435 981 719
(j) 0.000 004 72

2.6 Write the following numbers in the normal way (e.g 3 300):

(a) 3×10^{3}
(b) 2×10^{-2}
(c) 6×10^{1}
(d) 76×10^{-3}
(e) 3.54×10^{0}
(f) 9.73×10^{8}
(g) 5.23×10^{-7}
(h) 3.2185×10^{-4}
(i) 6.9836×10^{5}
(j) 3.5×10^{-2}
(k) 8.54×10^{7}
(l) 1.25×10^{-1}

You key 3.4×10^{-9} into a calculator by pressing

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[3] [.] [4] [$\times 10^{n}$] [−] [9]

2.7 Do the following calculations on your calculator.

(a) $(3.0 \times 10^{8}) \div (6.6 \times 10^{-7})$
(b) $(3.0 \times 10^{8}) \div (3 \times 10^{-2})$
(c) $(3.0 \times 10^{8}) \div (2.3 \times 10^{2})$
(d) $(3.0 \times 10^{8}) \div (5 \times 10^{-11})$

3 Re-Arranging Equations

Whatever is done to one side of an equals sign must be done to the other also. Take, for example, the equation:

$$a = b + c$$

a is the subject. To make b the subject, one must look at what is done to b and do the inverse to both sides. In the above equation, c is added to b, so b is made the subject by subtracting c from both sides of the equals sign:

- Subtracting c: $a - c = b + c - c$
- Simplifying the right hand side: $a - c = b$
- Writing b as the subject: $b = a - c$

Addition and subtraction are inverse operations.

Multiplication and division are inverse operations.

Powers and roots are inverse operations.

Example 1 – Make y the subject of $x = 2 \times y + z$

The last operation on y is the addition of z, so subtract z from both sides:

$$x - z = 2 \times y$$

y is multiplied by 2, so divide both sides of the equation by 2:

$$(x - z)/2 = y$$

Example 2 – Make g the subject of $5\sqrt{g} = h + j$

Divide by 5:

$$\sqrt{g} = (h + j)/5$$

Square both sides:

$$g = (h + j)^2/25$$

3.1 Rearrange the following equations to make the variable in brackets the subject:

(a) $p = mv$ (m)
(b) $Q = It$ (I)
(c) $v = s/t$ (s)
(d) $F = ma$ (a)
(e) $W = mg$ (m)
(f) $M = Fd$ (d)
(g) $V/R = I$ (R)
(h) $P/I = V$ (P)
(i) $v = f\lambda$ (λ)
(j) $\rho = m/V$ (V)

3.2 Rearrange the following equations to make the variable in brackets the subject:

(a) $E = mgh$ (m)
(b) $P_1V_1 = P_2V_2$ (P_2)
(c) $v^2 = u^2 + 2as$ (a)
(d) $\sin(c) = 1/n$ (n)
(e) $V_\mathrm{p}/V_\mathrm{s} = N_\mathrm{p}/N_\mathrm{s}$ (N_s)

3.3 Make v the subject of the following equation:
$E = \frac{1}{2}mv^2$

3.4 If $u = 0$, make t the subject of the following equation:
$s = ut + \frac{1}{2}at^2$

3.5 Make $\sin(r)$ the subject of the following equation:
$$n = \frac{\sin(i)}{\sin(r)}$$

3.6 Make x the subject of the following equation:
$10(x + y) = 5(x - y)$

3.7 Make λ the subject of the following equation:
$t = k/\lambda$

3.8 Make r the subject of the following equation:
$$F = \frac{kQ_1Q_2}{r^2}$$

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3.9 Make T the subject of the following equation:
$$r\left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^2}$$

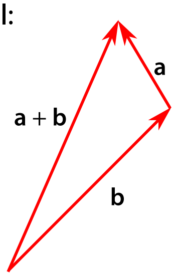
4 Vectors and Scalars

Scalar quantities have a magnitude (size) only, whereas vector quantities have a magnitude and a direction.

Vectors can be represented graphically as arrows. The length of the arrow indicates the magnitude of the vector. The direction of the arrow indicates the direction of the vector.

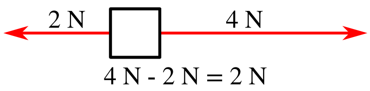
Quantity	Vector or Scalar?
Distance	Scalar
Time	Scalar
Displacement	Vector
Velocity	Vector
Acceleration	Vector
Speed	Scalar
Force	Vector
Gravitational potential energy	Scalar
Kinetic energy	Scalar
Momentum	Vector

When two vector quantities are added, the two arrows that represent the quantities are joined tip-to-tail:



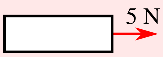
Subtracting a vector is the same as adding a vector pointing in the opposite direction.

If two vectors are in opposite directions, they add to give a vector with magnitude equal to the difference of the original vectors' magnitudes.



If two vectors are at right angles, the sum of their magnitudes can be calculated using Pythagoras' theorem.

4.1 In each example, state the size and direction of the unbalanced force acting on the object.

(a)


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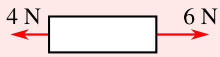

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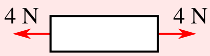

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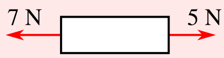

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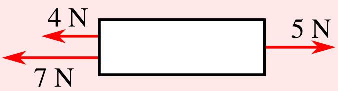
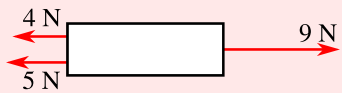
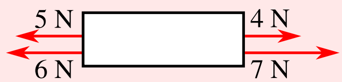


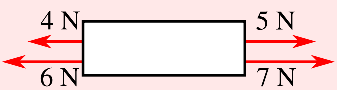
(f)


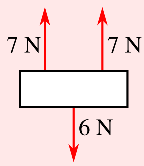
(g)

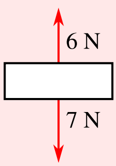

(h)


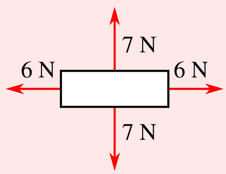
(i)


(j)


(k)


(l)


(m)


(n)


4.2 What is the resultant force on a racing car with 24.5 kN of driving force and 15.2 kN of opposing frictional forces (i.e. drag)?

4.3 What is the resultant magnitude of the displacement if a person walks north 5.00 km and east 4.00 km? i.e. How far away are they from the starting point?

4.4 Why is gravitational field strength a vector quantity?

4.5 Weight is a force. Force is a vector and so it has a direction. In what direction does your own weight point in these situations?

(a) not moving

(b) walking sideways

(c) walking in a circle

4.6 A stunt man drives a car out of the back of a moving lorry. For the stunt to work, the car must be moving at a velocity of -5.00 m/s the instant it has left the lorry. The lorry is travelling at a velocity of 25.0 m/s. What speed must the speedometer on the car reach before the car leaves the lorry?

4.7 Using a scale diagram, calculate the resultant force acting on a sailing boat when an easterly wind provides 2.50 kN of force, the tide provides 1.20 kN of force from the direction 30.0° more northerly than the wind.

4.8 A hiker walks 10.0 km east, 5.00 km south and 2.00 km west. Using a scale diagram, calculate his bearing from his start point. *[Note: bearings are given as angles where due north is 0° and the angle increases clockwise such that due east is 90.0°.]*

4.9 A kite is in equilibrium, so the total sum of the forces is equal to zero. On a vector diagram, the arrows representing the forces would form a closed loop. Three forces act on the kite; the force from the wind, the weight of the kite and the tension in the string. The wind produces a horizontal force of 70.0 N and an upward force of 50.0 N and the kite weighs 25.0 N. Use a scale diagram to find:

(a) the tension in the string;

(b) the clockwise angle the string makes to the horizontal.

5 Variables and Constants

Measurable quantities are either variables or constants. A variable is a quantity whose value can change. A constant is an unchanging quantity.

Commonly used constants include:

charge of the electron	-1.60×10^{-19} C
speed of light in a vacuum	3.00×10^{8} m/s

Some quantities *can* have different values (so they are variables), but within a particular experiment we do not expect their value to change. With these quantities, every effort should be taken to make sure their value remains as constant as possible. These are called control variables. Sometimes, deducing a value of a control variable and comparing this to an expected value is a useful way of testing the validity of the experiment. Common control variables include:

gravitational field strength at the surface of the Earth	9.81 N/kg taken as 10 N/kg at GCSE level
specific heat capacity of water	4 200 J/(kg °C)
speed of sound in air	330 m/s
refractive index of glass	1.50

In any experiment, the value of one quantity must be systematically changed in order to measure its effect on another quantity. The quantity that the experimenter chooses to change is called the independent variable.

The quantity whose value changes in response to the change of independent variable value is called the dependent variable.

Often, the independent variable and dependent variable values will be plotted on a graph so that the relationship between the two can be deduced and predictions can be made and tested.

5.1 Scientists wish to know the acceleration of a car as it rolls down a sloping ramp. They set the ramp at a certain angle and then release the car from different positions up the ramp, timing how long it takes to reach the bottom. There are several quantities that can be changed in this experiment. For each of the following, state whether it is a control variable, independent variable or dependent variable.

Variable	**Variable type**
Length of the ramp	(a)
Distance the car rolls	(b)
Duration of the car's motion	(c)
Mass of the car	(d)
Angle of the ramp	(e)
Surface material of the ramp	(f)

5.2 A sportsman wants to know the bouncing efficiency of a table tennis ball. He drops the ball from various heights and measures the maximum height the ball reaches after the first bounce. For each of the quantities listed in the table, state whether it is a control variable, an independent variable or a dependent variable.

Variable	**Variable type**
Size of ball	(a)
Material of ball	(b)
Height of ball before being dropped	(c)
Maximum height of ball after one bounce	(d)
Mass of ball	(e)
Material of surface onto which ball is dropped	(f)

6 Straight Line Graphs

To be able to correctly predict the effect of changing one variable on the value of another, physicists write equations. Part of the process of writing an equation requires the physicist to draw a graph, which reveals how one variable relates to another. When drawing graphs, it is common practice to

plot the independent variable on the x-axis (the horizontal axis), and the dependent variable on the y-axis (the vertical axis). Occasionally, it is more sensible to plot the variables on the axes the other way around. The equation for a straight line graph is:

$$y = mx + c$$

where y is the variable plotted on the y-axis, x is the variable plotted on the x-axis, m is the gradient of the straight line and c is the y-intercept.

At GCSE level, the relationship between two chosen variables is often linear, which means a graph of one variable versus another produces a straight line graph and the above equation works. Most equations at GCSE level can be written in the form $y = mx + c$.

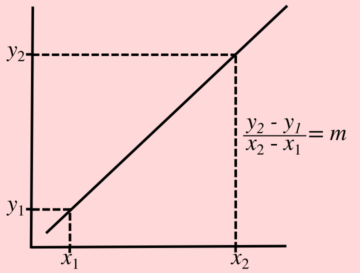
Example – If a student records every second how far something has travelled at constant speed, they can plot a graph distance on the y-axis and time on the x-axis. The gradient will be the speed.

6.1 A student wishes to measure the resistance, R, of a fixed resistor by varying the potential difference, V, across it and measuring the current, I, that flows through it. These quantities are related by $V = IR$. You might find it useful to re-write this relation as $I = (1/R) \times V$ The student plots V on the x axis.

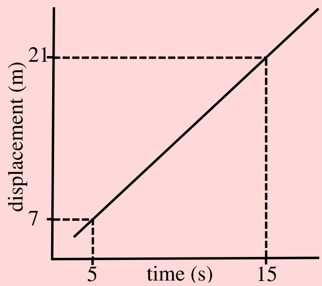
(a) What variable should be plotted on the y-axis?

(b) How can the resistance of the fixed resistor be determined from the graph?

The gradient of a straight line can be determined by considering two points on it:

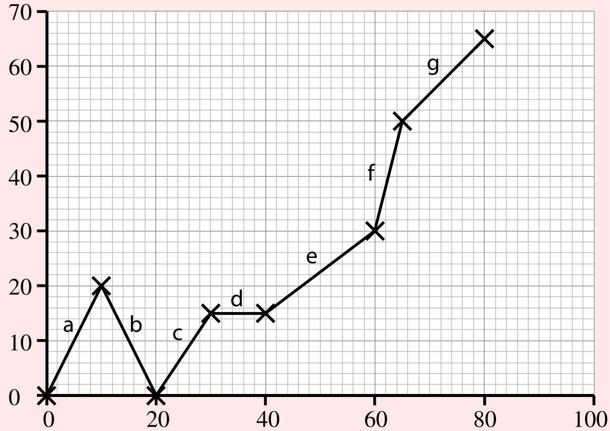


e.g.



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(21 - 7)\text{ m}}{(15 - 5)\text{ s}} = \frac{14\text{ m}}{10\text{ s}} = 1.4\text{ m/s}$$

6.2 For the following graph, calculate the gradient of the straight line sections labelled a, b, c, d, e, f and g.



6.3 Write the equation of a line which has a gradient of 2 if $y = 5$ when $x = 0$.

6.4 Write the equation of a line with gradient of 5, if $y = 7$ when $x = 1$.

6.5 Write the equation of a line with gradient of -8, if $y = 0$ when $x = 5$.

11/13

7 Proportionality

Physicists measure things, and then look for patterns in the numbers.

The most important pattern is called proportionality (also called direct proportion). If distance is proportional to time, it means that if the time doubles, the distance will double too. If the distance gets 10 times bigger, the time will get 10 times bigger as well. Mathematically, this is written as $s \propto t$.

> Example 1 – A particular resistor passes a 25 mA current when the voltage across it is 5.5 V. If voltage is proportional to current, what will the voltage be when the current is 60 mA?
>
> The new current is (60 mA / 25 mA) $= 2.4$ times larger than the old one. The new voltage will be 2.4 times larger than the old one: 5.5 V $\times\ 2.4 =$ 13.2 V.

Using a formula

If s is proportional to t then s/t will always have the same value. If we call this fixed value k, it follows that $k = s/t$, and that $s = kt$. We can use this information to answer questions. The formula method is much clearer if there are more than two quantities involved.

> Example 2 – A spring obeying Hooke's Law (its extension is proportional to the force) stretches by 14 mm when a 7.0 N load is applied. How far will it stretch with a 3.0 N load?
>
> We write force $= k \times$ extension, so extension $=$ force$/k$.
> $k =$ force$/$extension $= 7.0$ N$/14$ mm $= 0.50$ N/mm
> For a 3.0 N load, extension $=$ force$/k = 3.0/0.50 = 6.0$ mm.

> Example 3 – The energy transferred by an electric circuit in a fixed time is proportional to the voltage and also to the current $(E \propto V \times I)$. If the current is 3.2 A, and the voltage is 15 V and the energy transferred is 340 J. What current will be needed if we need to deliver 640 J using 12 V in the same time?
> The equation is $E = kIV$, so $k = E/(IV) = 340$ J$/(3.2$ A $\times$ 15 V$)$ $=$

7.08 J/(AV)
Re-arranging gives $I = E/(kV) = 640/(7.08 \times 12) = 7.53 = 7.5$ A (2sf)

7.1 A cyclist can travel 9.0 km in 30 minutes on level ground. Assume that their speed is constant.

(a) How far will they go in 120 minutes?

(b) How far do they go in 20 minutes?

(c) How much time will it take them to cover 27 km?

(d) How much time will it take them to cover 15 km?

7.2 One day, €1.00 is worth £0.83. On that day

(a) How many pounds would be needed to receive €200 when exchanging your money?

(b) How many euros could I get for £150 (to the nearest €)?

(c) A sandwich in a popular tourist city costs €6.50. How much is that in pounds (to the nearest penny)?

(d) A railway ticket costs £23.50. How much is that in euros(to the nearest cent)?

7.3 The UK minimum wage was £3.87 per hour for someone under the age of 18. Assume that your employer paid you this wage.

(a) How much did you earn for 20 hours of work?

(b) You worked 100 minutes a day after school. How much did you earn a day?

(c) How many hours would you have had to work to save £200?

7.4 The number of widgets made in a factory each week is proportional to the number of workers and the number of hours each worker works. When the factory employs 25 staff, each working 35 hours/week, $65\,400$ widgets were made each week.

(a) How many widgets would be made each week if 40 staff worked for 30 hours per week?

(b) If we need $130\,000$ widgets made each week, and the staff will work 42 hours/week, how many workers are needed?

7.5 The merchandiser at a warehouse sends stock to stores in proportion to their sales. She has 670 pairs of mauve trousers to dispatch. Her sales figures tell her that 124 pairs of trousers were sold in total last week, with the New Town branch selling 18 of them. How many pairs of trousers should she send to New Town?

7.6 A watch is set to the correct time at noon on 1^{st} January and put in a drawer. When it is checked at noon on 1^{st} February, it reads $11{:}51{:}20$. What did it read at $6{:}00$am on 24^{th} January?

Inverse Proportionality

The time taken on a journey is inversely proportional to the speed. If you double the speed, the time halves. If you only go at a tenth of the speed, it takes $10\times$ as long. We write this as $t \propto 1/v$, where v is the speed. In this case, $v \times t$ always has the same value.

Example 4 – The number of books printed each day is proportional to the number of printers owned, and inversely proportional to the number of pages in each book.
If $3\,000$ 300-page books can be printed in one day on 8 printers, how many 125-page books can they print on 6 printers in a day?

As books $\propto$ printers and books $\propto 1/$pages, books $= k \times$ printers$/$pages.
$k =$ books $\times$ pages$/$printers $= 3\,000 \times 300/8 = 112\,500$.
books $= k \times$ printers$/$pages $= 112\,500 \times 6/125 = 5\,400$.

7.7 A cyclist's journey to work takes them 32 minutes at 19 km/h. [*Hint: time* $\times$ *speed* $= 32 \times 19 = 608$.]

(a) How long would it take at 15 km/h?

(b) How fast would they have to go to reduce the time to 25 minutes?

7.8 An interest free loan for a luxury sofa takes 15 months to pay back at

£80 / month. Monthly charge $\propto$ number of sofas / duration of loan. What would the monthly charge be if I bought 3 sofas and paid for them over one year?

7.9 The current through a resistor is inversely proportional to its resistance. With a 330 Ω resistor, the current is 25 mA. What value of resistance is needed if you wish a 55 mA current to pass?

7.10 The braking force required to stop a car is inversely proportional to the time taken to stop it. If a 5 500 N force can stop the car in 8.0 s, how much force would be needed to stop it in 3.5 s?

$^{15}/_{20}$

Additional Proportionality Questions

7.11 Which two criteria must be met for a line graph to indicate direct proportionality between two quantities?

7.12 For each of the following equations state whether the two stated variables are directly proportional, inversely proportional or neither. [If there are other values in the question, they are kept constant.]

(a) $W = mg$ - W and m

(b) $pV = kT$ - p and V

(c) $p = mv$ - p and v

(d) $F = k\frac{Q_1 Q_2}{r^2}$ - F and r

(e) $T(\text{K}) = T(^\circ\text{C}) + 273$ - T(K) and T(°C)

(f) $a = 4\pi^2 r f^2$ - a and f^2

Mechanics

8 Speed, Distance and Time

When we study motion, distance is a scalar quantity that is equal to how far an object has moved. It is measured in metres in SI units. Other units include centimetres, inches, yards, miles and lightyears.

Time is central to the study of motion. It is measured in seconds in SI units. Other units include minutes, hours and days.

Speed is a scalar quantity that is equal to how far an object has moved divided by the time taken. It is measured in metres per second in SI units. Other units include miles per hour, parsecs per jubilee and feet per Julian year: Any speed unit using a distance unit divided by a time unit is valid.

The equation for average speed is:

$$\text{average speed} = \text{total distance} / \text{total time} \quad [v = s/t]$$

In the equation, physicists use v for speed and s for distance. These symbols are useful for more advanced mechanics. Always define your symbols.

Typical speeds are: Walking: 1.5 m/s Running: 3 m/s Cycling: 6 m/s

8.1 Use: speed = distance / time to calculate the missing values.

Distance (m)	Time (s)	Speed (m/s)
100	10	(a)
990	3.0	(b)
2.0×10^3	5.0	(c)
(d)	10	330
(e)	5.0	3.0×10^8
3 600	(f)	12
1.2×10^9	(g)	3.0×10^8
1.7×10^4	(h)	340

8.2 Work out the missing measurements from the following table, where each row is a separate question.

Average speed	Total distance	Total time
330 m/s	(a)	10.0 s
(b)	6.00 km	20.0 μs
3.00 m/s	45.0 m	(c)
(d)	40.1 km	24.0 hours
29.8 km/s	940 Gm	(e)
0.047 0 km/h	(f)	2 min 33 s
(g)	100 m	8.13 s

8.3 A train has an average speed of 100 kilometres per hour. Explain why the maximum speed could be different.

8.4 How far can you run in 15 seconds at an average speed of 8.0 m/s?

8.5 How long does a car take to travel 2.4 km at an average speed of 30 m/s?

8.6 A good long distance runner has an average speed of 5.5 m/s. How far would the runner go in 30 minutes?

8.7 The London-Glasgow shuttle takes approximately 60 minutes to fly a distance of 650 km. Estimate its average speed in m/s.

8.8 The wandering albatross can fly at speeds of up to 32 m/s (the speed limit on motorways!). One albatross was found to have flown 16 250 km in 10 days. Calculate its average speed in metres per second.

8.9 A cross-channel ferry travels at about 7 m/s. At the same average speed, how long would it take to cross the Atlantic Ocean, a distance of 6 700 km? Answer to the nearest hour.

8.10 How many kilometres is a 'light-year' – the distance travelled through space in a year by light at 300 million metres per second?

Additional Speed, Distance and Time Questions

8.11 At what speed does a bowler bowl a ball if it travels the length of the wicket to the batsman (20 metres) without bouncing in 0.45 s?

8.12 Concorde had a top speed of around $2\,180$ km/h; (that is, about twice the speed of sound in air, 340 m/s). Calculate its time to fly across the Atlantic Ocean from London to New York at this speed, a distance of $7\,600$ km.

8.13 A sock on the rim of a washing machine drum whilst it is spinning goes round in a circular path of radius 20 cm at a rate of 15 times per second. Calculate the speed of the sock in metres per second. Remember that the circumference of a circle, $c = 2\pi r$.

8.14 A marathon race is run over a distance of $42\,730$ metres. A top runner can complete the course in 2 hours 15 minutes. Calculate the average speed of the runner in metres per second.

8.15 Calculate the speed of a point on the Earth's equator as the Earth rotates once each day. The radius of the Earth is 6400 km.

8.16 Calculate the speed of the Earth in its orbit around the Sun if the radius of the orbit is 1.50×10^{11} m.

8.17 A delivery person starts their delivery round at $6{:}30$am. They travel a total distance of 5.00 km. At $7{:}53$am the delivery round ends. What was their average speed?

8.18 A police constable drives down a motorway with an average speed of 110 kilometres per hour. How far does the police constable travel in 15.0 seconds?

8.19 A groom is marrying his partner at $1{:}00$pm. The wedding venue is 6.00 km away from their house and the average journey speed is 40 kilometres per hour. What is the latest time he can leave his house in order to arrive on time?

9 Displacement and Distance

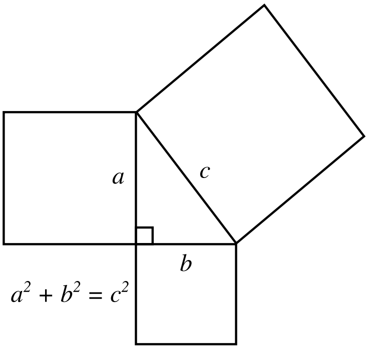
The straight line distance between an object's starting point and its end point - together with the direction - is called its displacement. The length of the path along which the object moves is the distance.

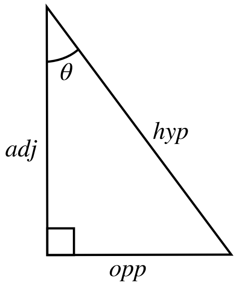
Displacement is a vector since it has a direction associated with it. Distance is a scalar; see Section 8.

Distance and displacement are both measured in metres (m) in SI units.

If an object moves in a circle, after one complete rotation, the displacement will equal zero and the distance will equal the circumference of the circle.

If an object is displaced in two perpendicular steps, the displacement can be calculated using Pythagoras' theorem and the direction can be calculated using trigonometry.





$$\sin\theta = \frac{\text{displacement opposite the angle}}{\text{displacement along the hypotenuse}}$$

$$\cos\theta = \frac{\text{displacement adjacent to the angle}}{\text{displacement along the hypotenuse}}$$

$$\tan\theta = \frac{\text{displacement opposite the angle}}{\text{displacement adjacent to the angle}}$$

9.1 A bus travels 500 m east, 250 m north, 500 m east and 250 m south.

(a) What distance has the bus travelled?

(b) What is the final displacement of the bus?

9.2 A climber climbs 50.0 m up a vertical cliff face, before being forced to climb back down 5.00 m so that she can find an alternative route to the top of the cliff. She climbs sideways to the left 10.0 m, then continues to climb 55.0 m to the top.

(a) What distance has the climber travelled?

(b) What is the magnitude (size) of her final displacement measured from her starting point?

9.3 An object is displaced by 60.0 m at a bearing of $60.0°$. If the object then moved 30.0 m due north, what is the final magnitude of displacement of the object from its origin?

9.4 A box is dropped from an aeroplane $2\,000$ m high travelling horizontally at 100 m/s. The box takes 20.2 s to hit the ground. While the box speeds up vertically, it continues at 100 m/s horizontally.

(a) What distance has the box travelled horizontally when it hits the ground?

(b) When the box hits the ground, what is the magnitude of the displacement of the box from the location it was released? i.e. how far is the box from the release point?

(c) What is the angle between the horizontal and the displacement of the box from the location it was released? Give your answer to 3 significant figures.

9.5 A bridge is a quarter of a circle of radius 20.0 m.

(a) What distance does a car travel whilst crossing the bridge?

(b) What is its displacement from the start to the end of the bridge?

9.6 A car is moving at a speed of 10.0 m/s. It is on a roundabout with a diameter of 50 m. After 23.56 s on the roundabout:

(a) What distance has the car travelled?

(b) How many turns of the roundabout has the car made?

(c) What is the magnitude of the car's displacement from where it entered the roundabout?

(d) Other than at the very start, $t = 0$ s, is it possible for the distance the car has traveled to equal its displacement at any point on the roundabout? Explain your answer.

$^{8}/_{11}$

10 Motion Graphs; Displacement–Time (s–t)

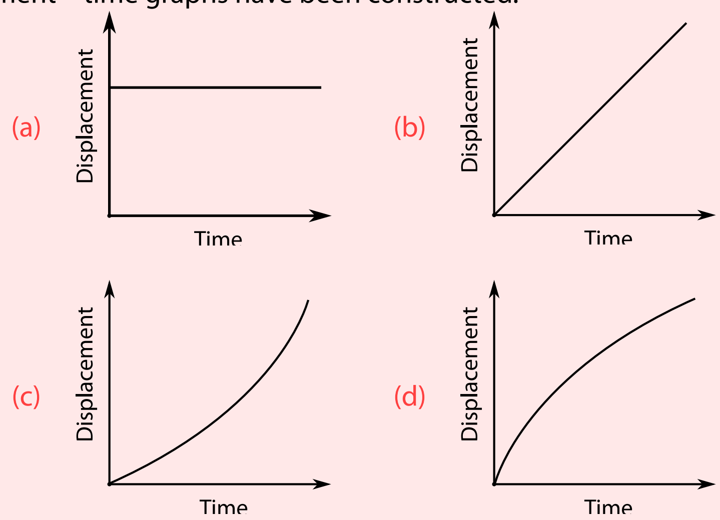
A displacement-time graph has displacement on the y-axis (the vertical axis) and time on the x-axis (the horizontal axis). The gradient of the line at any point is the velocity at that instant.
To review gradient calculations, see Straight Line Graphs - P14.

Take particular care of the unit for the gradient. It will be equal to the unit on the y-axis divided by the unit on the x-axis. For example, if displacement is measured in km on the y-axis and time in minutes on the x-axis, the gradient would have units of km per minute.

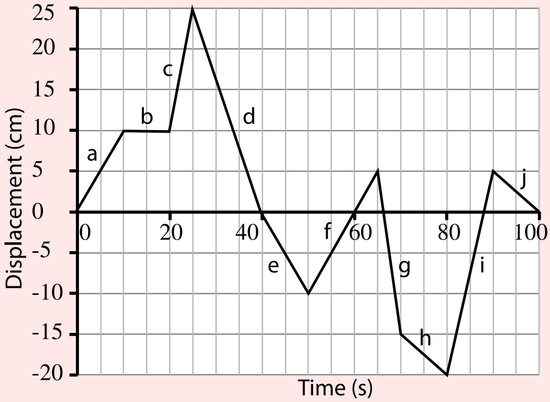
When displacement is on the y-axis, the direction of the displacement is equal to the direction of the velocity, unless the gradient has a negative value, in which case the direction of the velocity is opposite to the direction of the displacement.

When distance is on the y-axis instead of displacement, the gradient equals speed instead of velocity.

10.1 Describe the motions of the object for which the following displacement – time graphs have been constructed.



10.2 For the graph below, calculate the velocity for each labeled section a – j.



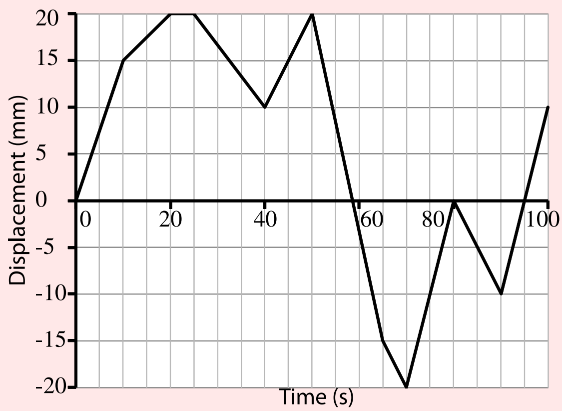
10.3 Considering the graph below:

(a) Between which times is the velocity most negative? Calculate the velocity between these times.

(b) Between which times is the velocity most positive? Calculate the velocity between these times.

(c) Between which times is the speed highest? Calculate the speed between these times.

(d) Between which times is the speed lowest? Calculate the speed between these times.



11 Acceleration

Acceleration means that there is a change of velocity – a change of speed or a change of direction of motion.

This could mean

- speeding up
  - when the acceleration is in the same direction as the motion
- slowing down (also called deceleration)
  - here the acceleration is in the opposite direction to the motion
- changing direction (a centripetal acceleration)
  - here the acceleration is at right angles to the motion

We measure acceleration in metres per second squared (m/s^2). An acceleration of 3 m/s^2 means that each second the velocity changes by 3 m/s.

$$\text{acceleration (m/s}^2) = \text{change in velocity (m/s)} / \text{time taken (s)}$$
$$a = (v - u)/t$$

When the velocity changes we use u for the velocity at the start, and v for the velocity at the end.

Example 1 – A car is travelling at 3.0 m/s. It accelerates at 2.5 m/s^2. How fast is it going 5.5 s later?

Change in velocity $= a \times t = 2.5$ m/s^2 $\times\ 5.5$ s $= 13.75$ m/s
New velocity $= 3.0 + 13.75 = 17$ m/s (2sf)

Example 2 – A car at 31 m/s stops in 6.8 s. Calculate the deceleration.

Acceleration = $(v - u)/t = (0\ \text{m/s} - 31\ \text{m/s})/(6.8\ \text{s}) =$
$(-31\ \text{m/s})/\ (6.8\ \text{s}) = -4.56$ m/s^2 so deceleration = 4.6 m/s^2 (2sf)
Here the velocity change is negative as the final velocity (0 m/s) is lower than the starting velocity (31 m/s), thus is a deceleration.

Example 3 – A car starts from rest. It accelerates backwards until it is reversing at 4.0 m/s. This takes 5.0 s. Calculate the acceleration.

Acceleration = $(v - u)/t = (-4.0\text{ m/s})/(5.0\text{ s}) = -0.80\text{ m/s}^2$.
The change in velocity is negative as the final velocity (-4.0 m/s) is lower than the starting velocity (0 m/s). However, although the acceleration is negative, this is not a deceleration as the car is speeding up (backwards).

11.1 Complete the table with the correct values. Each row represents a separate situation.

Acceleration (m/s^2)	**Velocity (m/s) after … s**						
	0.0	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**
3.0	0.0	3.0	(a)	9.0	(b)	(c)	18
5.0	0.0			(d)		(e)	(f)
7.0	3.0			(g)	(h)		(i)
-25.0	30.0			(j)	(k)		(l)
(m)	10.5		13.5		(n)		
(o)	45		36		27		(p)

11.2 In Q11.1(d), what would the velocity be 15 s after the start if the acceleration were maintained?

11.3 In Q11.1(o), at what time does the vehicle come to a stop?

11.4 A tennis ball is thrown in the air upwards at 15 m/s. If it is accelerating downwards at 10 m/s^2, what will its velocity be 2.0 s after it is thrown? (Remember to say how fast it is going and also which way.)

11.5 A rollercoaster speeds up from rest to 100 mph (45 m/s) in 1.2 s.

(a) Calculate the acceleration.

(b) The rollercoaster car then travels vertically upwards, and decelerates at $10\ \text{m/s}^2$. How much time passes before it is stationary (for a moment)?

11.6 A car starts from rest and reaches a speed of 40 m/s in a time of 8.0 seconds. Calculate its average acceleration.

11.7 Complete the table with the correct values. Each row represents a separate situation.

Starting velocity (m/s)	Final velocity (m/s)	Time taken (s)	Acceleration (m/s^2)
0.0	(a)	8.5	3.5
4.5	35	8.5	(b)
26	0.0	(c)	-6.7
(d)	5.0	1.2	-1.5
0.0	(e)	300	31

11.8 A certain make of car can reach 60 mph from rest in a time of 9.0 seconds. Calculate its average acceleration in m/s^2. [Note: 1 mph $=$ 0.45 m/s]

11.9 Calculate the change of speed of a train which accelerates for 9.0 seconds at a rate of $1.2\ \text{m/s}^2$ in a straight line.

11.10 In overtaking a lorry on a straight section of road, a driver increases speed from 50 mph to 70 mph in 5.0 s. [Note: 1 mph $=$ 0.45 m/s.] Calculate the acceleration in:

(a) miles per hour per second and;

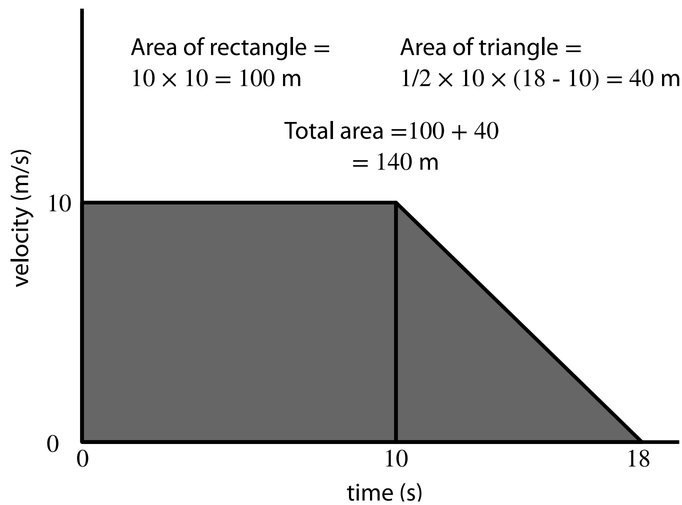
(b) metres per second per second.

12 Motion Graphs; Velocity–Time (v–t)

The displacement of an object moving with a constant velocity is equal to the product of the velocity and the amount of time the object is in motion.

To find the displacement when the velocity is changing, a velocity-time graph is needed. Normally, velocity is plotted on the y-axis (the vertical axis) and time is plotted on the x-axis (the horizontal axis).

The area under the line on a velocity-time graph is equal to the displacement of the object.



If the shape of the graph can be broken into simple geometric shapes, the total area under the line can be calculated by adding the areas of those shapes.

The area under a speed-time graph is the distance. Speed cannot be negative, and neither can the distance.

The area under a velocity-time graph is the displacement. Velocity can be negative if an object is moving backwards. The displacement can also be negative. An area beneath the x-axis has a negative value. An area above the x-axis has a positive value. Be careful when calculating the total displacement, when summing the displacements remember to include the $+$ and $-$ signs of the displacements.

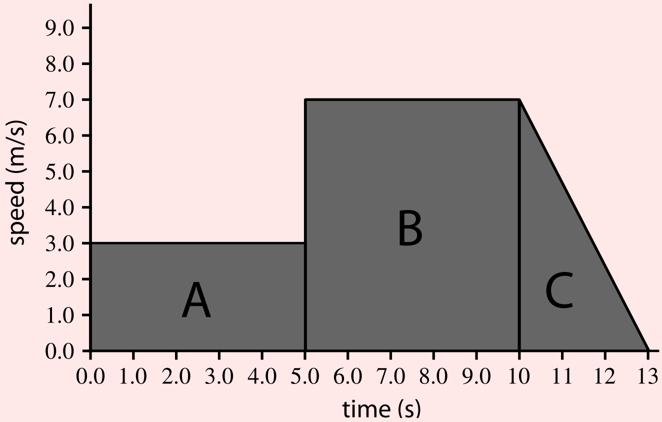
12.1 Using the following speed–time graph:

(a) calculate the distance travelled in A;

(b) calculate the distance travelled in B;

(c) calculate the distance travelled in C;

(d) calculate the total distance travelled.



12.2 Using the following graph:

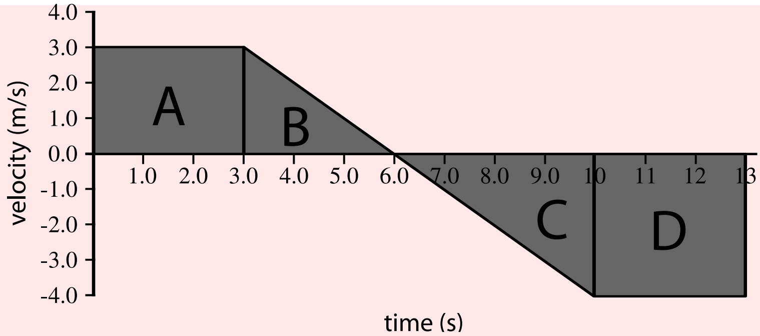
(a) calculate the displacement in A;

(b) calculate the displacement in B;

(c) calculate the displacement in C;

(d) calculate the displacement in D;

(e) calculate the total displacement.



12.3 For the motion described by the following speed–time graph, calculate:

(a) the distance moved in the first 10 s;

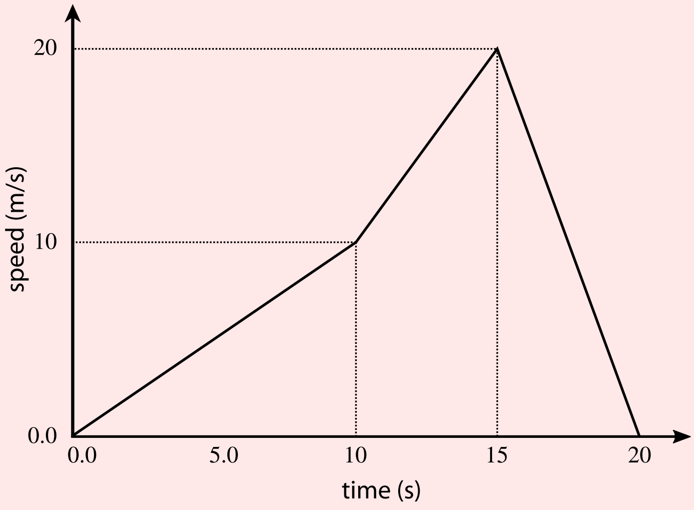
(b) the distance moved in the first 15 s;

(c) the total distance moved.

(d) The acceleration between 0 to 10 seconds.

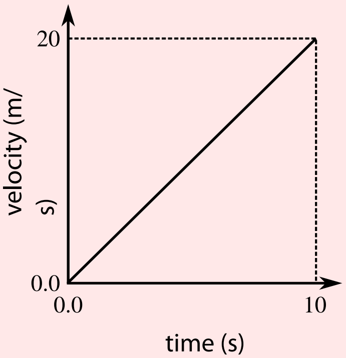
(e) The acceleration between 10 to 15 seconds.

(f) The acceleration between 15 to 20 seconds.

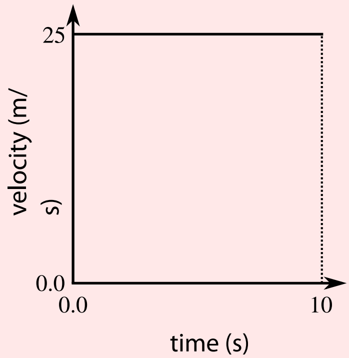


12.4 Calculate the displacement moved and the acceleration for the following velocity–time graphs.

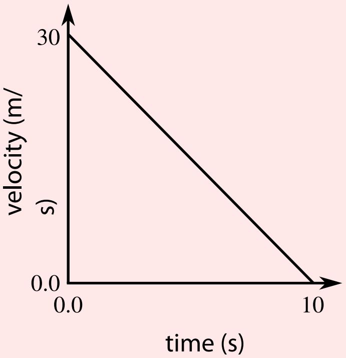
(a)



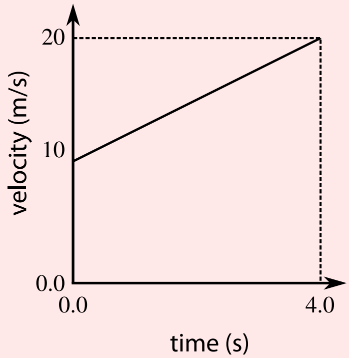
(b)



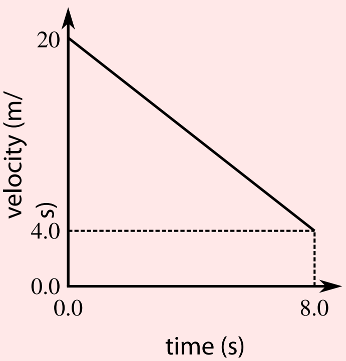
(c)



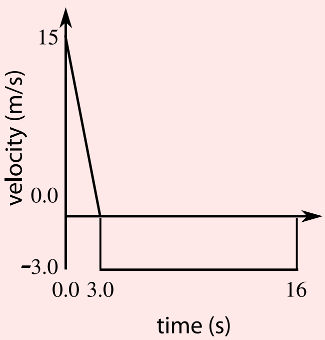
(d)



(e)



(f)

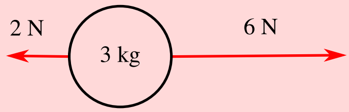


13 Resultant Force and Acceleration

The resultant force on an object is:

- the force left over after equal and opposite forces have cancelled out;
- the one force which would have the same effect as all of the forces;
- the vector sum of the forces on the object.

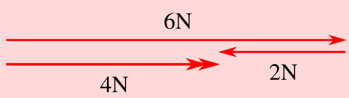
Example 1 – Calculate the resultant force on this object.



2 N force to left cancels out 2 N of the 6 N of the right force, leaving 6 N $-$ 2 N $=$ 4 N to the right left over.

Or you can answer: The two forces are $+6$ N and -2 N. Adding gives 4 N.

Or you can add the vector arrows 'nose to tail' to get a resultant 4 N answer:



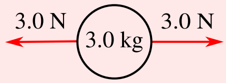
[A double arrow symbol here denotes a resultant vector.]

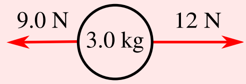
13.1 For these questions, refer to the diagrams that follow.

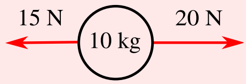
(a) Work out the strength and direction of the resultant forces for each object. *[Hint for (vi): draw the vectors nose to tail and think 'Pythagoras'.]*

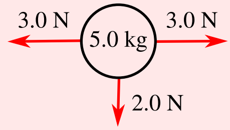
(b) Work out the strength and direction of the extra force which would need to be added in order to achieve equilibrium (zero resultant force) for each object.

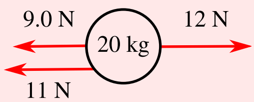
(c) Compare your answers to (a) and (b). What do you notice?

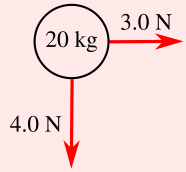
(i)


(ii)


(iii)


(iv)


(v)


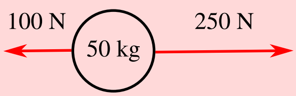
(vi)


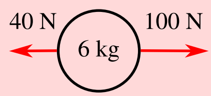
If you need more practice, turn back to Vectors and Scalars - P9 and try to balance the forces in Q4.9.

The acceleration of an object depends on the:

- resultant force acting on the object;
- mass of the object.

Example 2 – Which of these objects will have the greater acceleration?

(a)


(b)


(a) has resultant 150 N to the right, acting on 50 kg of mass. This means 150 N / 50 kg = 3 N/kg, i.e. 3 N acting on each kilogram.
(b) has resultant 60 N to the right, acting on 6 kg of mass. This means

60 N/6 kg = 10 N/kg, i.e. 10 N acting on each kilogram.
Therefore, object (b) will have the greater acceleration.

Formula:

$$\text{acceleration (m/s}^2\text{)} = \text{resultant force (N) / mass (kg)} \qquad a = F/m$$

Usually written:

$$\text{resultant force (N)} = \text{mass (kg)} \times \text{acceleration (m/s}^2\text{)} \qquad F = ma$$

13.2 Calculate the acceleration of each of the objects in Q13.1.

13.3 Complete the table. Each row represents a different question.

Resultant Force (N)	**Mass (kg)**	**Acceleration (m/s^2)**
(a)	810	6.7
(b)	430 000	2.6
2 000	65	(c)
(d)	10 g	9.8

13.4 A 100 g mass has weight of 1.00 N.

(a) If this is the only force on the mass, what is its acceleration?

(b) What would be the weight of a 300 g mass in the same gravitational field?

(c) If the weight is the only force on the 300 g mass, what is its acceleration?

A resultant force in the direction of motion speeds an object up.
A resultant force opposite to the direction of motion slows it down.
Zero resultant force means that the object keeps a steady velocity.

13.5 Complete the table. Each row describes a different object which has two forces acting upon it– one forwards (in the direction of motion), one backwards. Define forces and accelerations acting forwards as positive. Is each object speeding up or slowing down?

Force (N)			Mass (kg)	Acceleration (m/s^2)
Forwards	Backwards	Resultant		
58	16	(a)	5.6	(b)
90	145	(c)	22	(d)
(e)	350	(f)	120	+6.7

13.6 What unbalanced force acts on a 70 kg mass if it accelerates at 1.6 m/s^2?

13.7 What is the acceleration of a 10 kg mass which has no unbalanced force acting on it?

13.8 A 1 200 kg vehicle is accelerating along a straight road at 3.0 m/s^2. What is the magnitude of the unbalanced force acting on it?

13.9 What force must I apply to a mass of 3.0 kg to accelerate it at 4.0 m/s^2 on a horizontal surface if

(a) there is no friction and;

(b) there is friction of 4.0 N?

13.10 The thrust generated by a rocket engine is equal to the mass of propellant burnt each second multiplied by the exhaust velocity of the gas. The Space Shuttle (with booster rockets and external tank) had a total mass of 2 040 000 kg at launch. In this question we shall assume that the exhaust velocity of the gas was 3 000 m/s.

(a) How much propellant would have to be burnt each second in order for the spacecraft to just lift off?

(b) How much propellant would have to be burnt each second in order for the spacecraft to accelerate upwards from the launch pad at "3g" (i.e. 30 m/s^2)?

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Additional Resultant Force and Acceleration – on-line

13.11 A trained athlete runs a race. Her legs produce a constant force forwards. Her mass is 80 kg. The instant she starts to run, her acceleration is $3.0\ \mathrm{m/s^2}$.

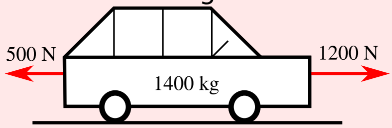
(a) What is the maximum accelerating force provided by her legs?

(b) After a short time, her acceleration has fallen to $1.0\ \mathrm{m/s^2}$ yet she is not tired. At this instant, what is the air resistance?

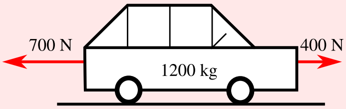
13.12 What is the frictional force if a 3.0 kg mass accelerates along a horizontal surface at $2.5\ \mathrm{m/s^2}$ when acted on by a pulling force of 10 N?

13.13 A 1 500 kg car accelerates at $1.5\ \mathrm{m/s^2}$ along a horizontal road. If the frictional forces acting against the car's motion total 1 000 N, what driving force is exerted on the road by the car's wheels?

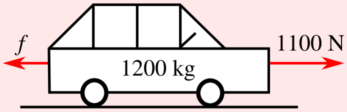
13.14 The 1 400 kg car below is travelling along a flat, straight road. Calculate the resultant force acting on the car. Calculate the car's acceleration with these forces acting on it.



13.15 The 1 200 kg car below is travelling along a flat, straight road.Calculate the resultant force acting on the car. Calculate the car's acceleration with these forces acting on it.



13.15



13.16

13.16 The car above is travelling along a flat, straight road. Its mass is 1 200 kg and it is accelerating forward at $0.50\ \mathrm{m/s^2}$. Calculate the resultant force acting on the car and the size of f, the friction force.